Pigou, Becker and the Regulation of Punishment-Proof Firms

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Abstract: We study the use of fines and inspections to control production activities that create external damages. The model contains a continuum of firms, differing in their compliance costs, so that only high-cost firms evade the regulations. Modifying the usual Pigou rule for taxing externalities to account for costly inspections, the external damage from the marginal evader's activities should exceed the expected fine by an amount equal to the resources expended to reduce the number of evaders a unit. According to Becker's classic work on crime and punishment, however, these resources can be minimized by raising the fines to very high levels, while reducing costly inspections. We argue that the modified Pigou rule does not hold under such a policy, because it distorts capital markets. Firms caught evading the regulation will be bankrupted by the fines, and the possibility that they will not fully repay investors lowers their expected cost of capital. Investors will lend to all firms at an interest rate above the social opportunity cost of capital, to compensate for the risks of bankruptcy. The paper investigates the optimal choice between the Becker approach of high fines and few inspections, versus keeping fines low enough to eliminate capital-market distortions, in which case the modified Pigou rule holds. High inspection costs favor the Becker approach. In some case, welfare can be improved over the Pigou optimum with an equilibrium under which some regulation-evading firms risk bankruptcy, whereas others choose capital stocks low enough to eliminate such risks.

1. Introduction

Economic agents engage in a wide variety of activities that generate external effects. For example, drivers impose congestion costs on others when they use public roads and may endanger others by driving recklessly; homeowners may anger neighbors by listening to loud music or by allowing their property to deteriorate; and firms may generate hazardous waste as part a byproduct of production or expose their workforce to unnecessary health risks by not talking sufficient care in designing their factories. Society responds to such situations by attempting to regulating behavior and by punishing those who violate the established rules. Sometimes the behavior is criminalized (it is illegal to dump hazardous waste), while in other instances attempts are made to internalize the external damages (toll roads). In the economics literature there are two classic treatments of the issues that surround such activity, due to Pigou (1920) and Becker (1968), but the analyses differ in focus, and they offer solutions that have starkly different tones. Our goal in this paper is to offer a new approach that unifies the messages of Pigou and Becker by showing that the optimal policy prescription for activities that generate external costs can take on either form, and identifying the conditions that determine which form it takes.

Pigou addressed the issue of externalities in *The Economics of Welfare.* An externality arises whenever the social cost of an activity differs from the private cost. Pigou's solution was to add a set of taxes to the price mechanism that would force individuals to internalize the full social costs. Thus, the Pigouvian solution is to set a tax which equals the marginal damage associated with the activity. If the external cost of the activity is low, the Pigouvian tax will be low; whereas activities that generate large external costs will be subject to large Pigouvian taxes. In this sense, the policy prescription proposed by Pigou is one in which the punishment fits the crime. Although Pigou (1954) acknowledged that there will be informational problems both in designing the optimal tax scheme and implementing it, the issue of compliance played no role in his analysis. In addition, Pigou's analysis did not emphasize the illegal nature of non-compliance.

In contrast, the illegal nature of non-compliance is at the center of Becker's (1968) analysis of such issues in "Crime and Punishment: An Economic Approach." Becker was interested in the question of how society should go about enforcing laws that criminalize activities that generate external costs. He focused on laws that are enforced by random inspection. The key policy parameters are the probability of detection, adjusted by increasing the rate of inspection, and the level of the fine imposed on those convicted of non-compliance. Becker's goal was to find the optimal policy; the one that minimizes the cost of the illegal activity.[1] He argued that because detection is costly while fines are nearly costless, the fine should be raised all the way up to the full wealth of the perpetrator. This policy enables the regulation to be enforced with a low probability and low cost of detection. It is important to note that in Becker's world, it is optimal to set the fine at a very high level, regardless of the costliness of detection and regardless of the extent of the external cost of the activity. Thus, with Becker's policy prescription, the size of the punishment does not necessarily fit the crime – those found guilty of non-compliance are always driven to the edge of bankruptcy regardless of the extent of the external damage.

It is clear that economists were uncomfortable with the counter-intuitive policy prescription of drastically high fines and low audit rates put forth by Becker. In fact, this finding is sometimes referred to as the "Becker conundrum" because we rarely observe such harsh punishment, even though the argument in its favor is clear and compelling.[2] Since 1968, over 200 articles have been published on the economics of enforcement, with many targeted at conquering the Becker

[1] Becker recognized the need to correct marginal incentives. In fact, in the early part of his paper, he derived the optimal fine for a fixed inspection rate, showing that in the first-best outcome, the expected fine should be set equal to the harm (as noted by Polinsky and Shavell 2000, this result actually dates back to Bentham 1789). However, Becker's focus was on enforcement. In particular, he argued that the existence of enforcement costs ensures that the marginal conditions that define the first-best outcome will not be satisfied. His solution of a high fine coupled with a low audit rate was designed to minimize the distortions created by such costs.

[2] In a survey of the literature on enforcement, Polinsky and Shavell (2000) provide a proof that the optimal fine is set at its upper limit when offenders are risk-neutral. Comparing this result with actual practice, they argue for higher fines. "Substantial enforcement costs could be saved without sacrificing deterrence by reducing enforcement effort and simultaneously raising fines."

conundrum.[3] In contrast, the robustness of Pigou's main result is rarely questioned.[4] Extensions have tended to focus on problems with implementation or complications that arise when Pigouvian taxes co-exist with other taxes.[5]

In this paper we argue that for certain regulations, Becker's analysis is too narrow, in the sense that it does not take into account the full implications of high fines. In particular, when firms must borrow or rent capital to produce, but face regulations that are imperfectly enforced, high fines may distort their choice of inputs and create inefficiencies in factor markets. The reason for this is that high fines alter the effective cost of capital that firms face, and they also affect the willingness of investors to lend to firms that may engage in black-market activities. The costs of these distortions must then be balanced against the benefits from reduced detection costs associated with higher fines. Below we develop a model that explicitly takes these potential factor-market distortions into account and show that it is optimal to enforce some regulations with moderate fines and likely detection, while others require with severe fines. In particular, we show that when enforcement costs are low, it is optimal to adopt a "Pigouvian approach" to regulation with relatively low fines that never drive violators to bankruptcy. In contrast, when enforcement costs are high, a "Beckerian approach" is optimal, with fines that not only bankrupt some or all firms but seize some or all of the assets that are involved in the illegal activity.

[3] For example, harsh fines are not optimal if agents are risk averse (Polinsky and Shavell 1979), because high fines impose an additional risk-bearing cost. In addition, if illegal activities can take on different gradations, it is optimal to impose moderate fines on less serious violations, thereby maintaining sufficient marginal incentives to deter more serious offenses (Sandmo 1981). Other approaches concern the optimal treatment of self-reported violations (Innes 1999), the structure of the criminal justice system (Rubenfeld and Sappington 1987; Malik 1990; Andreoni 1991; and Acemoglu and Verdier 2000), and heterogeneity among offenders (Babchuck and Kaplow 1993).

[4] For important exceptions, see Buchanan (1969), Carlton and Loury (1980, 1986) and Kohn (1986). In addition, as is well known, Coase (1960) argued that when transactions cost are low, Pigouvian taxes will not be needed to reach an efficient outcome. He argued that as long as property rights are well defined, economic agents will be able to agree to the first-best outcome and split the surplus that will be created by eliminating distortionary behavior.

[5] The double dividend literature stresses that in addition to correcting behavior, Pigouvian taxes generate revenue for the government. This creates a secondary benefit by allowing the government to reduce other taxes in the economy that may be creating distortions, but the modern literature has emphasized flaws in this argument (see, for example, Bovenberg and de Mooij 1994, Fullerton and Metcalf 1998, or Fullerton, Leicester and Smith 2010). The problems associated with collecting the information required to implement a Pigouvian tax (for example, measuring the true social cost) were stressed Baumol (1972) and a steady stream of related work has followed.

An interesting feature of our analysis is that there exists some fines and inspection rates under which the only equilibria contain ex ante identical black-market firms that make different investment decisions: some choose to be overleveraged, meaning they are bankrupted if caught evading the regulations, whereas others have sufficient assets to pay the fine. Moreover, fines and inspection rates that generate these equilibria may be optimal.

In the next section, we sketch the basic framework of our model and provide the intuition for our key results. As we explain, there are three regimes of enforcement. In the first regime, fines are below the level that would drive a violator to bankruptcy, so that regulation is similar in tone to Pigou's original design. In this regime, which is fully characterized in Section 3, firms use an efficient mix of inputs, and the price of capital for the relevant industry equals the economy-wide opportunity cost of capital. In the second regime, which has a tone consistent with Becker, the optimal fine exceeds each black-market firm's ability to pay so that the government is forced to seize some of the assets owned by investors if the firm is convicted of non-compliance. When the fine is this high, we show in Section 4 that these firms over-employ capital and that the cost of capital for the industry exceeds its efficient level. These factor market distortions are a direct result of the severity of the punishment scheme that the government uses for enforcement and they generate additional costs to enforcing the regulation that have not been discussed in previous work on this topic. The third regime, which occurs for intermediate-valued fines, is the one where some but not all black-market firms are overleveraged. We call such equilibria "hybrid equilibria" and also describe them in Section 4.

Starting from the optimal Pigou equilibrium, the remaining sections analyze the welfare effects of moving to a hybrid or Becker equilibrium. Section 5 presents an expression for the welfare change from marginal changes in the fine and inspection rate, and Section 6 uses it to sign the welfare effects of a small move into the hybrid regime from the optimal Pigou equilibrium.

Section 7 considers a large move, to a Becker equilibrium. Although it may be optimal for the fine to bankrupt firms, the optimal fine need not involve seizing all of the firm's assets.

2. Framework and Intuition

Our model consists of a perfectly competitive industry, in which firms borrow capital from a competitive factor market, and a government. The firms face a government-imposed regulation of some sort, and compliance is costly. We assume that the cost of compliance varies across firms so that, in equilibrium, some firms choose to comply with the regulation, whereas other firms operate in the black market, risking detection and punishment, by ignoring the regulation. Neither the government nor potential investors can observe the firm's behavior (or its cost of compliance) without monitoring, so investors cannot condition their investment decisions on the legal status of the firm. The government enforces the regulation by randomly inspecting firms and fining violators. The government's goal is to set the regulation parameters (the inspection rate and the fine) in a manner that maximizes social welfare.

The novel feature of our analysis is that we take into account the impact of regulation on factor market decisions. Thus, we begin by examining the firm's choice of inputs. We assume that each risk-neutral firm produces a single unit of output (x) using two inputs, entrepreneurial activity (e) and capital (k), according to a production function, $q(e,k)$, with neo-classical properties. Capital is provided by investors, who are promised that after all markets clear, they will be repaid the principal of the loan along with interest at rate r. The principal consists of the unit of capital, which does not depreciate, and the cost of a unit of entrepreneurial activity is normalized at one.

The firm's ability to repay investors will be determined by its choice of inputs, its behavior with respect the law, and the size of the potential punishment. In particular, since entrepreneurial assets are the residual claimants, the firm will have assets of $p + k$ to pay principal, interest and fines, where p denotes the price of the product. If a black-market firm chooses an input mix that ties up its liquidity, then the fine is paid first and any remaining assets go to investors. If investors receive less than the principal and interest owed to them, the firm is said to be "bankrupt." Black-market firms that leave themselves with more liquidity may be able to pay large fines without bankruptcy.

The firm's input decision is depicted in Figure 1 with the convex curve representing the unit isoquant. For law-abiding firms, the isocost curve is a straight-line with a slope of -*r* and, as is usual, the firm minimizes costs at the tangency of the two curves. These firms always use an efficient mix of inputs if r equals the social opportunity cost of capital (denoted by r^*). Things are somewhat different for black-market firms; for them, the slope of the isocost curve will also depend on the regulation parameters. To see this, note that for any given level of the fine, F, there exists a critical level of capital, $k_F \equiv \frac{p-F}{r}$, such that a black-market firm that selects $k \geq k_F$ will be bankrupt by the fine if caught violating the law. This firm will realize that it's effective cost of capital changes at k_F. If the firm selects $k \leq k_F$, then it will carry sufficient liquidity to pay the fine and fully repay investors regardless of circumstances. In this range, the firm's effective cost of capital is the same as it is for a law-abiding firm, *r*. However, if the firm selects $k_B \geq k_F$, it will fully compensate investors when it successfully evades the law, but it will be able to pay investors only the amount $p - F - k_B$. In this case, a one unit increase in k_B increases the firm's liability to investors by *r*: $(1+r)k_B - \{(p+k_B) - F\} = F - (p - rk_B)$, if fined. If we use π to denote the inspection rate, then the marginal cost of capital for black-market firms is $(1-\pi)r$ for $k \geq k_F$.[6] A higher inspection rate lowers this marginal cost because it increases the probability that the interest on additional investment is effectively paid by the government through reduced fine payments, at no additional cost to the firm. As a result, the isocost curve facing a black-market firm is kinked, with a

[6] To be precise, for any given F, the expected cost of producing one unit of output is $e + rk + \pi F$ when $k \leq k_F$ and $e + \pi p + (1-\pi)rk$ when $k \geq k_F$. Thus, the marginal cost of capital is *r* for $k \leq k_F$ and $(1-\pi)r$ for $k \geq k_F$.

slope of -r for $k \leq k_F$ and -$(1-\pi)r$ for $k \geq k_F$. Since the kink occurs at k_F, it will never be optimal for the firm to use the level of capital that leaves it exactly bankrupt when fined.

Figure 1 illustrates the case where the black-market firm is indifferent between choosing low and high levels of k. In other words, the kinked isoquant has two tangencies with the indifference curve, one on each side of the kink. More generally, when the when the fine is low, the kink occurs at a low value for k, and it is optimal for the firm to operate on the steep portion of the isocost curve, at a point such as A in Figure 1. However, when the fine is high, the kink occurs at a low value of k, and the firm will operate along the flatter portion of the isocost curve, at a point such as B. In other words, a high enough fine raises the marginal cost of capital from r to $(1-\pi)r$, causing the firm to increase its capital from k_A to k_B, and insuring bankruptcy in the event of an inspection.

In designing the optimal policy, the government then faces a trade-off. If it uses low fines and frequent inspection, which we refer to as Pigouvian regulation, firms will use the proper mix of inputs; while there may be significant enforcement costs, factor markets will operate efficiently. The other option, which we refer to as Beckerian regulation, is to use severe fines with a low rate of inspection, but this will lead firms to distort their mix of inputs. This option has low enforcement costs, but this benefit must be weighed against the cost associated with inefficiency in the factor markets. Below we show that the former solution is optimal when enforcement costs are low, and the latter is optimal when these costs are high.

The other point that we wish to emphasize in this section is that when black-market firms select their inputs, they effectively decide whether to expose themselves to potential bankruptcy, and this has important implications for the industry's ability to attract investors. In particular, under a Becker regime, firms that evade the regulation and are fined will be unable to repay investors in full. In a sense, these firms are punishment proof, at least at the margin. This implies that the government will seize some of the assets owned by investors. And, since investors cannot distinguish between law-abiding and black-market firms, they will anticipate the risk of seizure and demand higher capital rents from all firms in the industry. In equilibrium, the price of capital to the industry will exceed its economy-wide opportunity cost. This is another type of production distortion that accompanies high fines: the price of capital to the industry will be inefficiently high. None of these issues arise under Pigouvian regulation.

3. Pigouvian Regulation

We are now ready to begin our formal analysis, which we divide into three parts. First, in this section, we confine our attention to situations in which the government finds it optimal to use low or modest fines, so that firms minimize costs at a point such as A in Figure 1. In the next two sections, we consider the case of severe fines, and, finally, in Sections 6 and 7 we compare the two outcomes to find the globally-optimal enforcement mechanism.

Our perfectly competitive firms face two decisions – what input mix to use and whether to abide by the law. We denote the unit cost function by $c(r)$.[7] The firms are identical in all aspects except one, the cost of compliance. We use α_i to denote firm *i*'s cost of complying with the regulation and we assume that this firm-specific parameter is drawn *after the firm enters the market* from a continuous distribution function, denoted by $G(\alpha)$. Thus, the total cost of production for a law-abiding firm is $c(r) + \alpha$.

Alternatively, a firm may choose to operate in the black market where it saves the cost of compliance but risks detection and punishment. The probability of detection π and the fine F are the same for all firms. Thus, the expected total cost of producing and operating in the black market is $c(r) + \pi F$. It follows that we can divide firms into two groups: those with low compliance costs that

[7] The unit cost function includes only payments for capital and entrepreneurial effort. Note that with low or modest fines, the unit cost function is completely determined by r. This will not be the case in section 4, when the enforcement parameters will play a role in determining the effective cost of capital.

earn higher profits by abiding by the law (that is, firms with $\alpha \leq \pi F$) and the remainder, which earn higher expected profits by operating in the black market.

To complete the model, we now provide a description of the timing of decisions. In the initial stage, ex ante identical firms decide whether to enter the market. In stage two, α is revealed and each firm makes its input and compliance decisions. In particular, they decide on the mix of entrepreneurship and capital and sign contracts with investors. In stage three, production occurs and the product market clears. In stage four, the regulatory authority randomly inspections firms, detects non-compliance, and assesses fines, which must be paid immediately. Finally, in the last stage, investors are paid. The crucial assumption here is that the government collects fines *before* investors are paid. If the fine is set at a high level, then there may not be sufficient assets available to repay the investors if the firm is detected cheating.

We solve the model by backwards induction. The solution the firm's compliance decision is as described above: a risk-neutral firm with $\alpha = \alpha^* \equiv \pi F$ is just indifferent between compliance and non-compliance; all firms with $\alpha < \alpha^*$prefer to operate legally; and all firm with $\alpha > \alpha^*$prefer to operate in the black market.

We turn next to the firm's entry decision. Since the firms do not know their value of α before entry, their expected profits from production are given by

(1) $$E\Pi(p) = \int_0^{\alpha^*} [p - c(r) - \alpha] dG(\alpha) + \int_{\alpha^*}^{\infty} [p - c(r) - \pi F] dG(\alpha) - S,$$

where S is the sunk cost of entry. For any given set of enforcement parameters (F and π), there is a unique value of p at which expected profits are zero. For all higher p, all firms enter and there will be excess supply in the product market; for all lower p, no firm produces. Solving $E\Pi(p) = 0$ for p yields the market-clearing price:

(2) $$p = c(r) + \int_0^{\alpha^*} \alpha dG(\alpha) + \pi F[1 - G(\alpha^*)] + S.$$

We assume that the government also collects revenue from consumers by imposing a sales tax of t on this good, so that the price paid by consumers for each unit is $q \equiv p + t$. The assumption here is that while some firms evade the regulation, all firms pay the tax. For example, a regulation concerning a production process may be evadable, while no good possibilities exist for selling the product without paying a sales tax.[8]

On the demand side of the product market, the representative consumer has the following quasi-linear utility function:

(3) $U(x, q) = E - q + v(x) - h(x_n),$

where E denotes the consumer's lump-sum income, $x_n \equiv [1 - G(\alpha^*)]x$ denotes the output produced by black-market firms, and $h(x_n)$ is the external cost created by that output.[9] Income E consists of an endowment of the numeraire good, plus a government transfer financed by tax revenue and fines. We assume that $h(\cdot)$ is increasing and convex in x_n. The consumer treats x_n as fixed and chooses x to maximize utility. Thus, x satisfies the following first order condition,[10]

(4) $v'(x) = q = p + t$

Summarizing the product market, the producer price of output, p, is determined by the free-entry condition and is given by (2). Total output, x, is determined by the sales tax t and the solution to the consumer's maximization problem, given by (4). Since each firm produces one unit of output, x also denotes the equilibrium number of firms.

We now turn to the capital market, where we assume that investors obtain capital at the economy-wide rate (opportunity cost) of r^* and supply it to the firms in this industry, demanding an interest payment of r. In the Pigouvian equilibrium, firms that evade the regulation choose to carry

[8] For analyses of the welfare effects of black-market activities undertaken to evade taxes, see Davidson et al. (2005, 2007).
[9] Production by law-abiding firms creates no external costs because these firms comply with the regulation.
[10] We assume that E is large enough that (4) is satisfied for all relevant q.

enough liquidity to repay investors fully, in which case all firms face the interest rate $r = r^*$. We next describe the condition that must hold for firms to carry this level of liquidity.

If black-market firms choose a relatively low level of capital (as depicted by A in Figure 1), their expected costs are $c(r^*) + \pi F$; whereas the higher level of capital (as depicted by B in Figure 1) results in expected costs of $c[(1-\pi)r^*] + \pi p$.[11] Note that, as described in the previous section, the higher level of capital entails a lower effective cost of capital and leads to a lower payment by the firm when caught evading the regulation (the fine bankrupts the firm, so they simply turn over all of their revenue, *p*, to the government). For the lower level of capital to be optimal for the firm, as required for a Pigou equilibrium, it must lead to lower or the same expected costs, which occurs when

(5) $$\pi(p - F) \geq c(r^*) - c[(1-\pi)r^*]$$

Thus, bankruptcy will not occur in equilibrium if (5) is satisfied by the government's chosen regulation parameters. We refer to (5) as the "Pigou constraint."

In a Pigouvian equilibrium, all law-abiding and black-market firms use the same efficient mix of *e* and *k* in production, so that factor markets are not distorted in any way. The behavior of the private sector in any Pigouvian equilibrium is completely characterized by (2) with $r = r^*$, (4), and the cut-off value $\alpha^* \equiv \pi F$.

We now turn to the government's problem of optimal enforcement. The government imposes this regulation because non-compliance generates the external cost of $h(x_n)$. In addition to *h,* the government must also be concerned about the resources that it devotes to enforcement. This cost is given by $p_a \pi x$, where p_a denotes the cost of inspecting one firm and πx is the total number of

[11] At point B in Figure 1, the firm pays e_B to entrepreneurs, rk_B to capital owners when not inspected (which occurs with probability $1-\pi$) and $p - F$ to capital owners when inspected (which occurs with probability π). Thus, expected production costs at B are $e_B + (1-\pi)rk_B + \pi(p-F)$. In addition, the firm faces an expected fine of πF. Summing to get total expected costs, we obtain $e_B + (1-\pi)rk_B + \pi p = c[(1-\pi)r] + \pi p$.

inspections that are carried out. The government's goal is to choose π, t and F to maximize social welfare (W), which is given by

(6) $$W = v(x) - h(x_n) - x[c(r^*) + \int_0^{\alpha^*} \alpha dG(\alpha) + p_a\pi + S].$$

We assume that lump-sum transfers are available to balance the government budget.

The government's problem is to select the policy variables π, F and t to maximize (6) subject to (5) and the market equilibrium conditions. This leads to the following Lagrangian

(7) $$v(x) - h(x_n) - x\left[c(r^*) + \int_0^{\alpha^*} \alpha dG(\alpha) + p_a\pi + S\right] + \lambda[\pi(p - F) - c(r) + c[(1-\pi)r]],$$

where λ is the Lagrange multiplier. This problem can be simplified by noting first that the sales tax does not enter into any of the equilibrium conditions other than (4). Thus, the government can control output directly through t. Maximizing (7) over x yields the following first-order-condition

(8) $$v'(x) - h'(x_n)[1 - G(\alpha^*)] - \left[c(r^*) + \int_0^{\alpha^*} \alpha dG(\alpha) + p_a\pi + S\right] = 0.$$

If we use (4) to substitute for $v'(x)$, (2) to substitute for p, and then solve for t, we obtain

(9) $$t = [1 - G(\alpha^*)][h'(x_n) - \pi F] + p_a\pi.$$

We show below that when inspections are costly, optimal enforcement implies that $h'(x_n) > \pi F$, so that the expected fine falls short of the marginal damage created by black-market firms. Equation (9) the indicates that the government should set the sales tax to make up for this difference: since $1 - G(\alpha^*)$ is the probability that any given unit of output is produced by a black market firm, $h'(x_n) - \pi F$ is the marginal damage not paid for by the firm, and $p_a\pi$ is the inspection cost per unit of output, the right-hand-side of (9) is the residual external damage per unit of output associated with the optimal enforcement mechanism imposed on firms.

We next maximize (7) over π and πF, obtaining the following first-order-conditions[12]

(10) $-xp_a + \lambda(p - rk_B) = 0$

(11) $h'(x_n)g(\alpha^*) - x\alpha^* g(\alpha^*) - \lambda\big[1 - \pi\big(1 - G(\alpha^*)\big)\big] = 0$

where $g(\alpha^*) = G'(\alpha^*)$, and $k_B \equiv k[(1-\pi)r]$ denotes the cost-minimizing amount of capital used by firms when the marginal cost of capital is $(1-\pi)r$ (that is, at point B in Figure 1). From (10), $\lambda = \frac{p_a x}{p - rk_B} > 0$; which implies that the constraint in (5) always binds. Intuitively, if there is any slack in the constraint, the standard Becker argument applies – that is, the government can increase F and lower π holding πF constant and increase Social Welfare. With πF constant, there will be no change in the market outcome, and with fewer inspections, the government will save on enforcement costs.

If we now use (10) to eliminate λ in (11), we obtain our condition that defines the optimal expected fine under Pigouvian regulation:

(12) $h'(x_n) - \pi F = \frac{p_a[1-\pi(1-G(\alpha^*))]}{g(\alpha^*)(p - r^* k_B)}$.

If enforcement is costless ($p_a = 0$), then the marginal compliance cost ($\alpha^* = \pi F$) should be set equal to the marginal damage. This minimizes the social cost per unit of output. From (8), the optimal sales tax would then be zero. This policy generates the first-best allocation, which is the standard Pigouvian result.

With positive enforcement costs, the Pigouvian condition must be modified, and the first-best outcome can no longer be achieved. As the expected fine increases, black-market firms will be tempted to move into the bankruptcy region. To counteract this, the government must increase the inspection rate, but this increase is now costly. Thus, if the government wants to avoid bankrupting

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12 Alternatively, we could maximize (7) over π and F, but using π and πFas the policy parameters leads to a more straightforward analysis.

violators, it will have to moderate the punishment. From (12), the optimal cut-off α^* is below marginal damage by a term that is increasing in inspection costs.

To better understand optimality condition (12), rewrite it as follows:

(13) $$h'(x_n)g(\alpha^*) = \alpha^* g(\alpha^*) + p_a \frac{d\pi}{d\alpha^*},$$

where

(14) $$\frac{d\pi}{d\alpha^*} = \frac{1-\pi(1-G(\alpha^*))}{p-r^*k_B} > 0.$$

If the government increases the expected fine πF by a unit, the equality between the expected fine and marginal compliance cost α^* implies that α^* must rise. The left-hand-side of (13) captures the resulting welfare gain: $g(\alpha^*)$ is the measure of firms that leave the black market and $h'(x_n)$ measures the reduction in external damage that results from their compliance with the regulation. This gain comes at a cost, which is on the right-hand-side of (13): the first term is the cost of compliance for those firms that leave the black market, and the second term captures the increase in inspection costs that result from increasing the inspection rate. As we showed above, with optimal enforcement, Pigou constraint (5) must hold with equality. The derivative $d\pi/d\alpha^*$ is the marginal change in the inspection rate π needed to maintain this equality as the marginal compliance cost α^* rises. With $\alpha^* = \pi F$, $d\pi/d\alpha^*$ is also the marginal rate of substitution between the inspection rate and the expected fine.

Optimal enforcement requires that the marginal benefit of a reduction in black-market output be set equal to the marginal cost (see Figure 2, where the marginal benefit and cost schedules are depicted for a given output *x*, with the reduction in black market activity, Δx_n, measured on the horizontal axis). Note further that if external damages are large, the government will want to increase the severity of its policy to deter non-compliance.

To understand the determinants of $d\pi/d\alpha^*$, substitute from the price equation (2) for p, using $r = r^*$ and $\alpha^* = \pi F$, and then rewrite the resulting equation as follows:

(15) $$\frac{d\pi}{d\alpha^*} = \frac{\pi[1-\pi(1-G(\alpha^*))]}{\alpha^*-L_B},$$

where L_B is the deadweight loss that would be incurred if a firm chose the profit-maximizing inputs under the return $r(1-\pi)$:

(16) $$L_B = \pi r^* k_B - [c(r^*) - c((1-\pi)r^*)] = [e_B + r^* k_B] - c(r^*).$$

In words, this deadweight loss equals the excess of the cost of inputs e_B and k_B, evaluated at r^*, over minimized cost $c(r^*)$. Equation (15) is a first-order differential equation for the function $\pi(\alpha^*)$.

The deadweight loss L_B depends on how much the subsidy rate πr^* distorts capital investment. The usual triangle approximation of deadweight loss is,

(17) $$L_B = \frac{1}{2}\pi r^*[k_B - k_A] = \frac{1}{2}\pi^2 r^* k_A \varepsilon,$$

where $k_A = k(r^*)$ and ε is the elasticity of demand for capital, evaluated at r^*. This approximation becomes exact when the capital demand curve is linear. Substitution from (17) into (15) yields

(18) $$\frac{d\pi}{d\alpha^*} = \frac{\pi[1-\pi(1-G(\alpha^*))]}{\alpha^*-\frac{1}{2}\pi^2 r^* k_A \varepsilon}$$

It may seem strange that the marginal cost of compliance depends positively on the capital distortions associated with overleveraged firms -- that is, firms that would be bankrupted by fines -- because there are no overleveraged firms in the Pigou case. But the presence of these distortions can be explained by noting from the binding Pigou constraint (eq. 5 with an equality) that when F rises, the required rise in π lowers production costs $c[(1-\pi)r^*]$ for overleveraged firms, which by itself makes the overleverage option more attractive and therefore makes a given marginal rise in π less effective in restoring indifference between k_A and k_B. The relative importance of this consideration rises with $k_A - k_B$, causing the required increase in π to rise. Deadweight loss is positively related to the same $k_A - k_B$.

The value of $\pi(\alpha^*)$ is determined by the differential equation given by (18), once initial conditions are specified. We know that $\pi(0) = 0$, but this alone does not determine $d\pi(0)/d\alpha^*$ because the numerator and denominator in (18) are both zero at $\alpha^* = 0$. Rather, we can use $\alpha^* = \pi F$, noting that as π goes to zero, F converges to a value determined by the binding Pigou constraint, given by (5) with an equality, and expression for p, evaluating (2) at $\alpha^* = 0$:

(19) $$F = p - r^* k_B = p - r^* k_A = e(r^*) + S,$$

where use is made of the observations that, at $\pi = 0$, no fines are paid and all firms evade the regulation. Substituting $\alpha^* = \pi F$ with this F into (18) and taking the limit as π goes to zero gives

(20) $$\frac{d\pi(0)}{d\alpha^*} = \frac{1}{e(r^*)+S}.$$

Thus, a rise in $e(r^*) + S$ lowers the initial value of $d\pi/d\alpha^*$, presumably leading to lower inspection costs at positive values of α^*; that is, Pigouvian regulation becomes more attractive. The basic idea is that the higher are fixed costs and entrepreneurial returns, the higher is the equilibrium price of output, and this higher price enables a given rate of compliance to maintained with a higher fine and lower inspection rate.

4. Becker and Hybrid Equilibria

We now consider enforcement policies that bankrupt at least some inspected black-market firms. Fines are high enough to bankrupt such firms when Pigou constraint (5) is reversed; that is,

(21) $$\pi(p - F) \leq c(r) - c[(1-\pi)r],$$

where r may now exceed r^* to compensate investors for bankruptcy. We refer to (21) as the "Becker constraint." When it holds with a *strict* inequality, black-market firms and legal firms will *always* use different amounts of capital to minimize their cost of production. In particular, as described in Section 2, black-market firms will choose a higher level of capital, because they realize that if they fined, the marginal capital will be costless. Maintaining the notation introduced in Section 3, we denote the amount of capital used by black-market firms as k_B. Firms that operate at this point cannot pay their debts when fined, a situation we have referred to as "overleveraged." Equilibria where all black-market firms are overleveraged are referred to as "Becker equilibria."

If the Becker constraint holds with equality, then black-market firms will be indifferent between points A and B in Figure 2, and it is possible to have an equilibrium in which a fraction of black-market firms, $\gamma < 1$, are overleveraged, with the remainder operating at A. We refer to such equilibria as "hybrid equilibria." In this case, only those inspected black-market firms that are overleveraged will be driven to bankruptcy by the fine. To summarize, $\gamma = 1$ in a Becker equilibrium, $\gamma \in (0,1)$ in a hybrid equilibrium, and $\gamma = 0$ in a Pigou equilibrium.

When the government inspects overleveraged black-market firms, it will now lay claim to some income owed investors in an attempt to collect the unpaid fines. These anticipated seizures will distort the capital market and lead to a higher price of capital for the regulated market. In equilibrium, the profits earned by investors from supplying capital to this industry must exactly offset losses associated with the expected seizures. The government inspects a particular firm with probability π and seizes $F - (p - rk)$ units of assets from that firm if it has not complied with the regulation. Since the fraction of firms that decide to operate in the black market is $1 - G(\alpha^*)$ and γ is the fraction of black-market firms that are overleveraged, it follows that expected seizures are given by $\pi\gamma[1 - G(\alpha^*)]\{F - p + rk_B\}$. As for expected profits, all law-abiding firms and a fraction $(1 - \gamma)$ of all black-market firms employ $k_A = k(r)$ units of capital, while the remainder employ k_B units. Thus, since the investors pay r^* for the capital, their expected profits from supplying capital to this industry at rate *r* are given by $(r - r^*)\{k(r)[G(\alpha^*) + (1 - \gamma)\big(1 - G(\alpha^*)\big)] + k_B\gamma[1 - G(\alpha^*)]\}$

in the absence of seizures. The equilibrium r is determined by the requirement that these expected profits equal expected seizures:

(22) $$(r - r^*)\{k(r)G(\alpha^*) + [1 - G(\alpha^*)][(1-\gamma)k(r) + \gamma k_B\} = \pi\gamma[1 - G(\alpha^*)]\{F - p + rk_B\}.$$

Since the right-hand-side of (15) is positive in a Becker or hybrid equilibrium, it must be the case that $r > r^*$ in any such equilibrium. Thus, capital is paid a premium in the regulated industry.

The fact that law-abiding and overleveraged black-market firms use different amounts of capital has implications for the compliance decision. A firm with a compliance cost of α faces a total cost of $\alpha + c(r)$ if it operates legally and an expected cost of $\pi p + c[(1-\pi)r]$ if it operates in the black market and uses k_B units of capital. Note here that since the fine bankrupts the firm, expected costs are the same as they would be if the fine equaled p, so that investors were left with no interest income in the event of an inspection. The maximum fine is the firm's total assets, $p + k$, but any rise in the fine above p reduces payments of principal to investors by the increase in the fine, resulting in no change in total cost. The firm with the marginal compliance cost has the same total cost in the legal and black markets:

(23) $$\alpha^* = \pi p - \{c(r) - c[(1-\pi)r]\}.$$

To complete the description of equilibrium for a Becker or hybrid equilibrium, we turn to the product market. As discussed in Section 3, when firms enter the market, they must forecast that their revenue will exactly equal their expected costs for the product market to clear. The counter-part of (2) is then

(24) $$p = c(r)\big[G(\alpha^*) + (1-\gamma)\big(1 - G(\alpha^*)\big)\big] + \pi F(1-\gamma)(1 - G(\alpha^*))$$
$$+ \{c[(1-\pi)r] + \pi p\}\gamma\{1 - G(\alpha^*)\} + \int_0^{\alpha^*} \alpha dG(\alpha) + S$$

Using (23), we can solve this expression for the market-clearing price:

(25) $p = c(r) + [\alpha^*\gamma + \pi F(1-\gamma)](1 - G(\alpha^*)) + \int_0^{\alpha^*} \alpha dG(\alpha) + S$

In a hybrid equilibrium, where (21) holds with equality, (23) gives $\alpha^* = \pi F$, as in the case in a Pigou equilibrium. Finally, output and the number of firms are determined, as in the previous section, by the demand side of the product market – in particular, (4).

5. Social Welfare in a Becker or Hybrid Equilibrium

Social Welfare in a Becker or hybrid equilibrium is now given by

(26) $W = v(x) - h(x_n) - x[\tilde{c}(r,\alpha^*) + \int_0^{\alpha^*} \alpha dG(\alpha)] - p_a \pi x,$

where $\tilde{c}(r,\alpha^*) \equiv [e(r) + r^* k(r)][G(\alpha^*) + (1-\gamma)\big(1 - G(\alpha^*)\big)] + [e_B + r^* k_B]\gamma(1 - G(\alpha^*))$ is the expected social cost of production (that is, the cost of capital is evaluated at its opportunity cost, r^*). The only difference between this welfare expression and the welfare expression for a Pigou equilibrium is that capital markets are now distorted, raising production costs: $\tilde{c}(r,\alpha^*) > c(r^*)$. With $r > r^*$, there are two capital market distortions: firms choosing $k_A \equiv k(r)$ underinvest, and firms choosing $k_B \equiv k[(1-\pi)r]$ overinvest. The deadweight losses for both underinvested and overinvested firms may be expressed in terms of the quadratic loss expression:

(27) $L_A = \frac{1}{2}(\frac{r - r^*}{r^*})^2 r^* k_A \varepsilon_A$ *and* $L_B = \frac{1}{2}(\frac{r^* - r(1-\pi)}{r^*})^2 r^* k_B \varepsilon_B$

Once we are in a Becker or hybrid equilibrium, the change in welfare from a marginal change in the fine and inspection rate reflect changes in these deadweight losses. In particular, differentiating (26), while using the sales tax to keep x fixed, gives the following marginal change in welfare per unit of x:

(28) $$\frac{dW}{x} = (h' - \alpha^*)gd\alpha^* - p_a d\pi$$

$$-\left\{[G + (1-G)(1-\gamma)](r - r^*)\frac{dk_A}{dr}\, dr\right.$$

$$\left. + \quad (1-G)\gamma(r^* - r(1-\pi))(\frac{dk_B}{d\big((1-\pi)r\big)}(rd\pi - (1-\pi)dr)\right\}$$

$$-(1-G)(L_B - L_A)d\gamma \quad + (\gamma L_B + (1-\gamma)L_A)gd\alpha^*$$

As shown, the changes in deadweight losses take place through changes in the marginal returns on capital, through changes in the share of all firms that choose the black market, and, in the case of hybrid equilibria, through changes in the share of black-market firms that choose to become overleveraged. It may be desirable to tolerate these deadweight losses if a given level of compliance can be achieved with lower inspection costs. The next two sections analyze the tradeoff in detail.

6. Is a Hybrid Equilibrium Better than the Pigou Optimum?

We now investigate the conditions under which welfare can be improved by moving from the Pigou optimum to a fine and inspection rate that cause some firms to be overleveraged; that is, the economy moves to a hybrid equilibrium. In this case, $\gamma = 0$, $r = r^*$ and $L_A = 0$ initially. Moreover, a marginal increase in *r* from r^* has no first-order effect on deadweight loss L_A. Thus, (28) becomes:

(29) $$\frac{dW}{x} = (h'(x_n) - \alpha^*)g(\alpha^*)d\alpha^* - p_a d\pi - (1 - G(\alpha^*))L_B d\gamma$$

To obtain more compliance using a smaller increase in the audit rate, we must now tolerate the deadweight losses associated with some overleveraged firms. The issue is whether the overall cost of additional compliance can be lowered in this way. Assuming linear capital demand curves (so that the quadratic loss expression is an exact measure of deadweight loss), we now prove:

Proposition 1. *Starting from the Pigou optimum, a small increase in the fine and inspection rate that causes some firms to become overleveraged is desirable (undesirable) if*

(30) $\frac{\alpha^*}{L_B} < (>) 1 + \frac{2p_a}{r^* k_A}(1-\pi)$

or, using the quadratic loss expression, if

(31) $\epsilon > (<) \frac{F/\pi}{[\frac{1}{2}r^* k_A + p_a(1-\pi)]}$

Proof. Start with the optimal (π, F), determined by (13) and (15):

(32) $h'(x_n)g(\alpha^*) = \alpha^* g(\alpha^*) + p_a \frac{d\pi}{d\alpha^*}$; $\quad \frac{d\pi}{d\alpha^*} = \frac{\pi[1-\pi(1-G(\alpha^*))]}{\alpha^* - L_B}$.

Then implement a perturbation in the inspection rate and fine that involves increasing α^* a marginal unit, but with a rise in π that is an amount δ less than the amount needed to remain in the Pigou regime:

(33) $\frac{d\pi}{d\alpha^*} = \frac{\pi[1-\pi(1-G(\alpha^*))]}{\alpha^* - L_B} - \delta.$

With black-market firms indifferent about becoming overleveraged in the hybrid equilibrium, the Becker constraint (21) holds with equality. Substituting from (25) for price *p* in this equality gives

(34) $\pi\left(\alpha^*(1-G(\alpha^*)) + \int_0^{\alpha^*} \alpha dG(\alpha) + S\right) - \alpha^* = (1-\pi)c(r) - c[(1-\pi)r],$

where use is made of the equality between α^* and πF in a hybrid equilibrium. We may differentiate (34) to find the change in r from r^* needed to keep black-market firms indifferent about becoming overleveraged, following the changes in the expected fine and inspection rate satisfying (33):

(35) $\frac{dr}{d\alpha^*} = \frac{\delta(p - rk_B)}{(k_B - k_A)(1-\pi)} = \frac{\delta(\alpha^* - L_B)}{(k_B - k_A)(1-\pi)\pi},$

where the second equality was previously derived, using the definition of deadweight loss given by (16), along with a binding Becker constraint (21). Differentiating the condition for capital market equilibrium, given by (22), we obtain the marginal effect of a rise in r from r^* on the fraction of firms that choose to become overleveraged:

(36) $$\frac{d\gamma}{dr} = \frac{k_A}{\pi(1-G(\alpha^*))(F-(p-rk_B))} = \frac{k_A}{(1-G(\alpha^*))L_B},$$

Assuming a linear demand curve, we may re-express (35) in terms of deadweight loss L_B:

(37) $$\frac{dr}{d\alpha^*} = \frac{r^*\delta(\alpha^*-L_B)}{2L_B(1-\pi)}.$$

Multiplying (36) and (37) together then gives

(38) $$\frac{d\gamma}{d\alpha^*} = \frac{r^*k_A\delta(\alpha^*-L_B)}{2(1-G(\alpha^*))(1-\pi){L_B}^2}.$$

At the Pigou optimum, we know that the welfare change given by (29) equals zero when the change in the inspection rate satisfies (29) with $\delta = 0$. In this case, there is no change in γ. Thus reducing $d\pi/d\alpha^*$ by a positive δ, thereby moving into a hybrid equilibrium, as described by (38), causes the welfare change given by

(39) $$p_a\delta - \frac{d\gamma}{d\alpha^*}L_B(1-G(\alpha^*)) = p_a\delta - \frac{r^*k_A\delta(\alpha^*-L_B)}{2(1-\pi)L_B}.$$

Thus, welfare rises (falls) if

(40) $$p_a - \frac{r^*k_A(\alpha^*-L_B)}{2(1-\pi)L_B} > (<)\ 0$$

Rearranging (40) proves (30), and (31) follows from substituting into (30) the expression for L_B given by (27), and using $\alpha^* = \pi F$. Q.E.D.

The need for a lower bound on the capital elasticity may seem counter-intuitive, because low elasticities imply low deadweight losses from the distorted investment decisions of overleveraged black-market firms, therefore suggesting greater potential welfare gains from moving into the hybrid region. The explanation is that a high elasticity decreases the sensitivity of the number of overleveraged firms to higher inspection rates and fines. In particular, (38) shows that $d\gamma/d\alpha^*$ declines with deadweight loss L_B. The reason is that both $d\gamma/dr$ and $dr/d\alpha^*$ and decline with a rise in L_B, as described by (36) and (37).

Proposition 1 also shows that it is desirable to induce some firms to become overleveraged if inspection cost p_a is sufficiently large, all else equal. The reason is that higher inspection costs increase the desirability of achieving a given compliance level with fewer inspections and higher fines. But to do so, the government must move the fine and inspection rate out of the Pigou region, causing at least some firms choose to become overleveraged.

Another implication of Proposition 1 is that overleveraged firms are desirable if the ratio of the fine to the inspection rate, $\frac{F}{\pi}$, is sufficiently low. Unlike p_a, this ratio is obviously endogenous, but it does depend on exogenous variables. In particular, if entrepreneurial income and fixed costs are relatively low, then it will not be possible to increase the fine much, given the inspection rate, before the fine must effectively come out of payments to investors, thereby bankrupting firms. By similar reasoning, Proposition 1 also shows that overleveraged firms are desirable if capital income $r^* k_A$ is relatively high.

7. Is a Becker Equilibrium Better than the Pigou Optimum?

Once we move to a Becker equilibrium, the impacts of the fine and inspection rate on the share of black-market firms that are overleveraged is no longer an issue, because all black-market firms are now overleveraged. In terms of our general expression for the welfare change, given by (28), the welfare term, $(1-G)(L_B-L_A)d\gamma$, disappears, and we can generally conclude that moving from the Pigou optimum to the Becker optimum will be beneficial if deadweight losses created by the move, L_B for black-market firms and L_A for legal firms, are sufficiently small, which will be the case for *small* capital elasticities. Thus, looking over all three types of equilibria, the global optimum will be a Becker equilibrium for sufficiently small capital elasticities.

Combined with the Proposition 1, we then find that starting from the Pigou optimum, inducing a small number of firms to become overleveraged may be welfare-reducing, while inducing all black-market firms to become overleveraged can then improve welfare. In other words, the welfare effects can be non-monotonic.

How small the deadweight losses from capital market distortions must be for a Becker equilibrium to be optimal will depend on the amount by which the marginal benefit of a higher expected fine exceeds the marginal compliance cost at the Pigou optimum, as measured by the term $(h' - \alpha^*)g(\alpha^*)$. Recall that this net marginal benefit would equal zero if there were no inspection costs, but high inspection costs can cause it to be large (recall eqs. 13 and 14). If it is large, indicating that inspection costs have greatly reduced compliance, then increasing the fine a lot can be desirable, even with sizable deadweight losses that result from the equilibrium becoming the Becker type.

Finally, if a Becker equilibrium is possible, then it may, but need not, involve a fine so high that all of the firm's assets are taken by the government, leaving the firm with no money to pay capitalists. Suppose, for example, that production uses one of only two techniques, distinguished by capital intensities (see Figure 3). Moving from the optimal Pigou equilibrium to a Becker equilibrium distorts production by causing black-market firms to choose the inefficient capital-intensive technique. But once the all black-market firms are choosing this technique, a further increase in the fine increases the required return further above r^* but has no effect on the choice of techniques by any firm. Hence, increasing F becomes socially costless, so F should be raised all the way to $p+k_B$.

On the other hand, suppose that we changed the model to one with three techniques (an indifference curve with two downward-sloping flat sections), where the middle technique is socially efficient; that is, it is picked at interest rate r^*. Then the optimum could be a Becker equilibrium, with $F < p+k_B$, where black-market firms choose the most capital-intensive technique, due to the effective capital subsidy they obtain by becoming overleveraged, while firms that comply with the regulation choose the efficient middle technique. Increasing the fine to $p+k_B$ might then not be desirable, if the resulting rise in the equilibrium interst rate r caused the legal firms switch to the least capital-intensive technique, thereby raising deadweight loss.

To conclude, low capital elasticities may justify fines that are high enough to bankrupt firms, but this bankruptcy need not entail fines that take all of a firm's assets.

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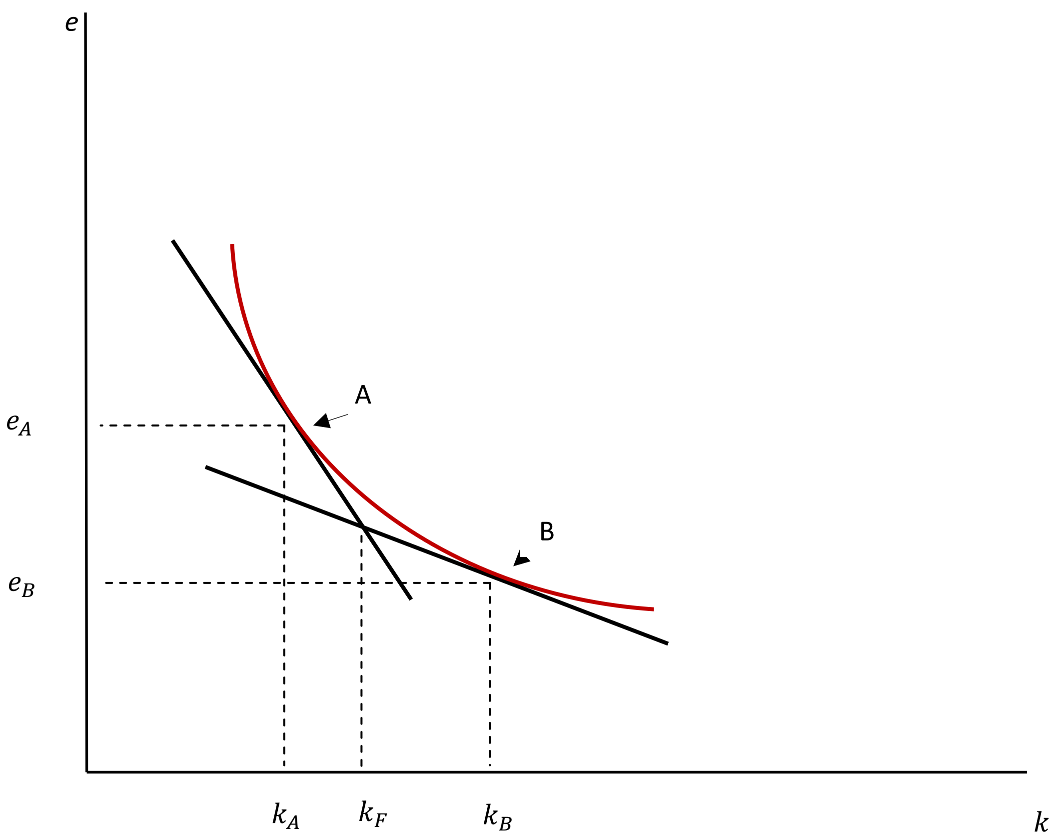

Figure 1: Choosing Inputs

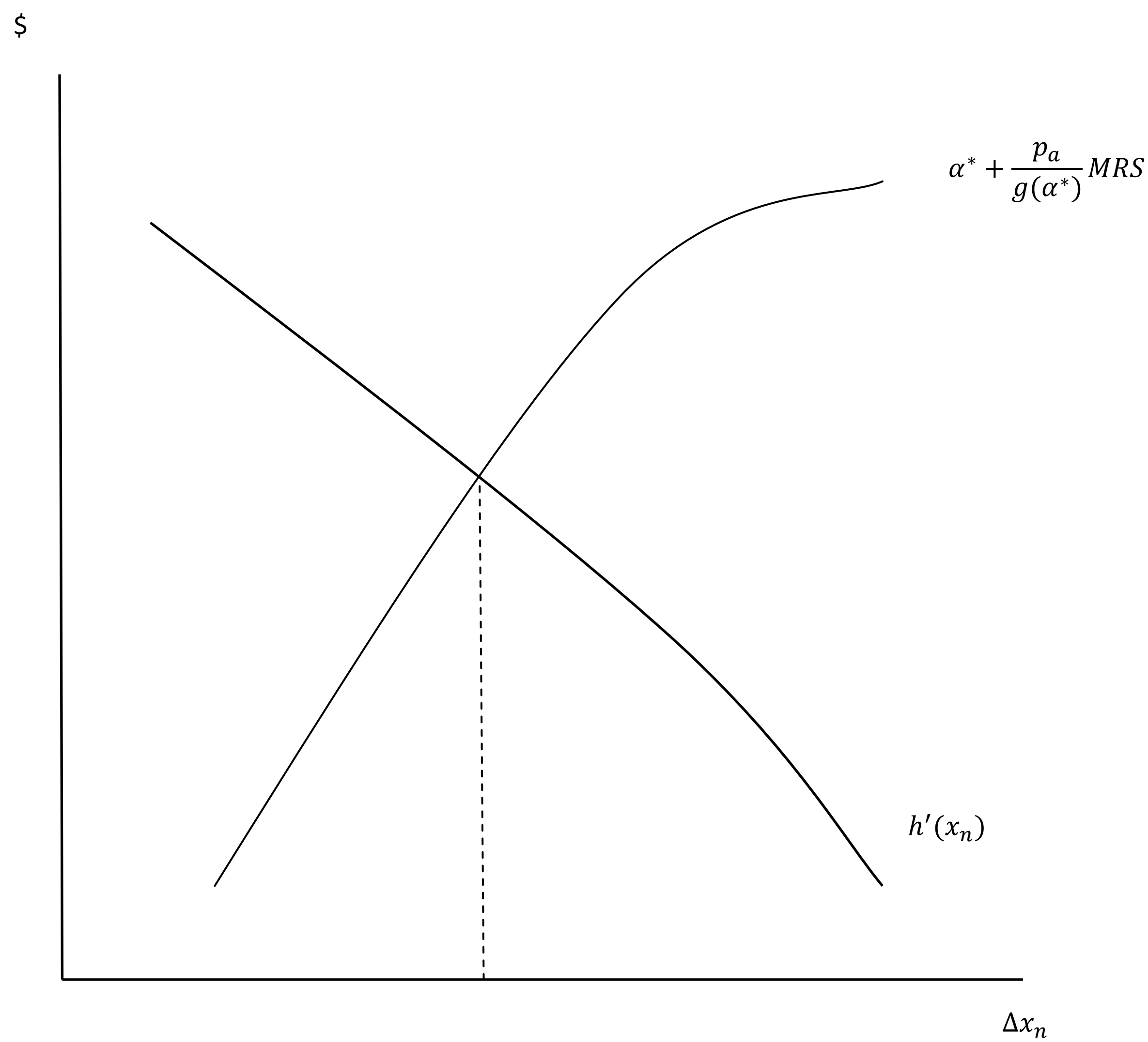

Figure 2: Optimal Pigouvian Regulation

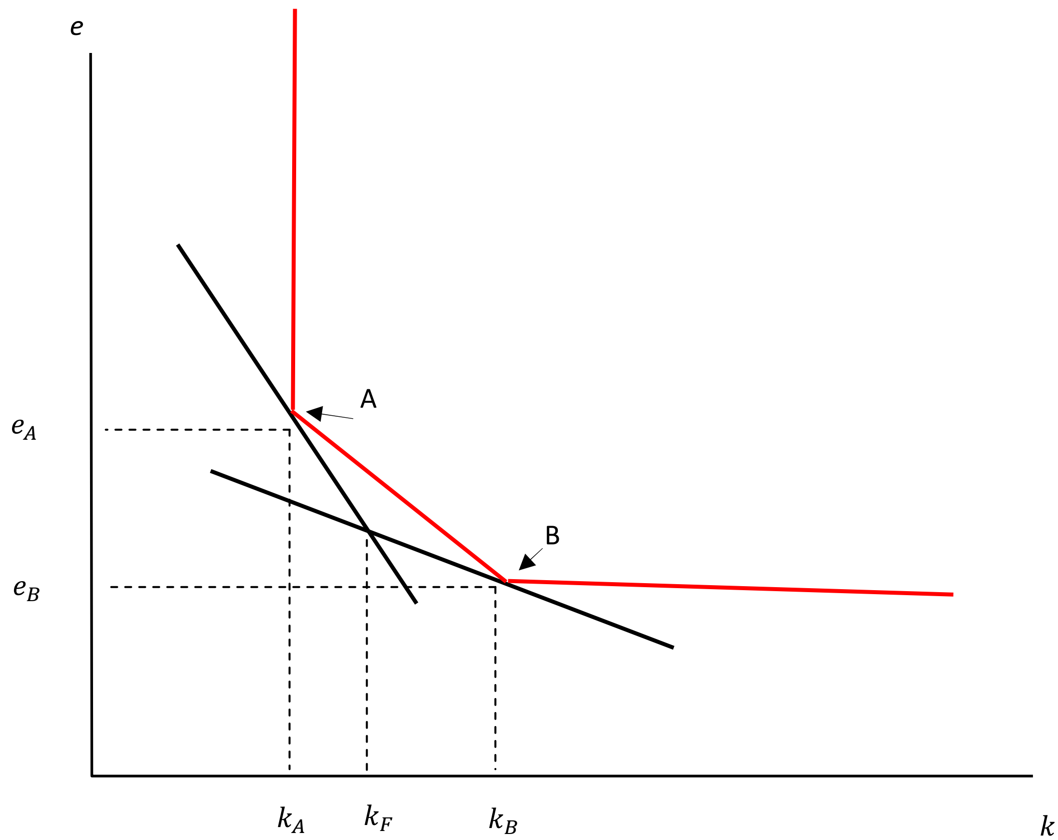

Figure 3: Two Production Techniques